CSci 435: Formal Languages and Automata

Name: Adam Roy

**Home Assignment 3: 90 points + 10 (optional)**

Q1. [10] For L = L(*aa*\*) from ∑ = {*a, b*}, find a regular expression for

Hint: Construct a DFA M’ s.t. L(M’) = using a DFA M whose L(M) = L.

Then, give a regular expression for .

The language L accepts any string that starts with a and is followed by 0 or many a’s

The Language LC accepts strings that don’t start with a and allows for a sequence of {a,b} that doesn’t have consecutive a’s. the regular expression would be (b + ab\*)\*

Q2. [15]

1. [10] By applying Theorem 4.1, find a minimal DFA that accepts L(*ab\*a*\*) ∩ L(*a\*b\*a*).

Hint: First, construct a DFA for the given language using Thm 4.1.

Then, minimize it to a DFA with 6 states.

A diagram of a flowchart

Description automatically generated

[5] Give the regular expression for the above language in 1) that is accepted by the DFA.

L1 = (ab\*a\*) -> a(b\*a\*)\*

L2 = (a\*b\*a) -> (a\*b\*a\*)a

The combined language would be a(b\*a\*)(a\*b\*a\*)a

Q3. [10] The family of regular languages is closed under arbitrary ***homomorphism***.

Prove or disprove h(L1 ∩ L2) =h(L1) ∩ h(L2) is a regular language where L1 and L2 are regular.

If the given claim is false, give a counter example to disprove it.

By Theorem 4.1 regular languages are closed under union.

Lc1 and Lc2 are regular so L1∩L2 = Lc1 U Lc2 is also regular meaning

h(L1∩L2) = h(Lc1) U h(Lc2) is also regular.

Q4. [15] Let L1 = {L(*b*\**abb*\*) and L2 = L(*bab*\*). Find the ***right quotient*** of L1 with L2, L1/L2.

1. [10] Let M be a DFA s.t. L(M) = L(L1).

By applying Thm. 4.4, construct a DFA M’ s.t. L(M’) = L1/L2.

L1/L2 = L{b}\*

A diagram of a circle with arrows

Description automatically generated

1. [5] Give a regular expression for the *right quotient* with L1, i.e. L(M’) = L1/L2.

The regular expression would be b\*a(a|b)\*

Q5. [10] If L is a regular language, prove that the language L2 = { *uv* | *u*∈ LR , *v* ∈L } is also regular.

Assuming that L is regular the reversal of language L is also regular therefore u and v are regular and since from theorem 4.1 we find that concatenation of regular languages is regular proving uv is regular.

From this we can determine that L2 is regular

Proof: assuming that L is defined by a regular expression P, PR | L(PR) = (L(P))R

Q6. [10] Define the Complementary OR (COR) of two languages by

COR (L1, L2) = { *w* | *w*∈ or *w*∈ }

Show that the family of regular languages is closed under the COR operation.

COR(L1, L2) = {w | w E Lc1} U {w E Lc2}

Assuming that L1 and L2 are regular languages we can assume that M1 and M2 are accepted by L1 and L2

Therefore {w | w E Lc1} U {w E Lc2} is regular and closed under union conditions and we can assume that COR(L1, L2) = {w | w E Lc1} U {w E Lc2}

Q7. [10, optional] The ***left quotient*** of a regular language L1 with respect to L2 is defined as:

L2/L1 = { *y* | *x*∈ L2 , *xy* ∈L1 }

Show that the family of regular languages is ***closed*** under the ***left quotient*** with a regular language.

Hint: Do NOT construct a DFA that accepts L2/L1 but use the definition of L2/L1 and the closure

properties of regular language.

Q8. [20] Pumping Lemma (PL)

1. [10] Prove that the language L = {*anbkcn* | *n* ≥ 0, *k* ≥ *n* } is ***not regular***.
2. Choose your *w,* s.t. |*w*| ≥ *m* > 0.
3. Clearly give your substrings and *x, y, z* after the partition of *w=xyz*, satisfying their requirements in PL. Explain your rationale to choose x*, y, z*.
4. Decide your number of pumping *i*.
5. Show that your *wi* = *xyiz* is not in L.

L contains the strings {abc, abbc, abbbc, … }  
proof by contradiction states that the language will be regular and according to pumping lemma there is a pumping length P which will be 5 so P =5; then there is a string S and we will make S = aabbbcc such that |S| >= P which is true 7 >= 5. Because that is true there exists a split of S; S = x,y,z such that

x = a y = abbb z = cc; with |xy| <= P 5 <= 5 True

and |y| > 0 also true and when i >= 0 the string xyiz element of L for i >= 0. If we consider the situation where i = 2; x y2 z = a abbbabbbcc which is not a string in the language L which proves that the language is not regular.

1. [10] Prove or disprove that L1 ∪ L2 is not regular language if L1 and L2 are not regular languages.

L1 U L2 contains the strings {aabbbcc, aabbbbcc} when considering a contradiction we assume that a language is regular and then by pumping lemma there exists a pumping length P such that P = 2; then  
|S| = P and when taking the string S = aabbbcc and when |S| >= P is true we can split ‘S’ into xyz again where x = a y = a z =bbbcc

|xy| <= P -> 2 <= 2 True

|y| > 0 -> 1 > 0 True

Continuing; for xyiz is the element of Lx for i >0; by taking i = 2 again we find that xy2z = a(a)2bbbcc

-> aaabbbcc

The above string does not fit in the language so the string S is not an element of L1 U L2 therefore L1 U L2 is not regular.